COMP 2805 — Assignment 3

Due: March 19. Due to renovations in Herzberg 4135, you have to submit the assignment in the main office of the School of Computer Science (Herzberg 5302). The office will be open until 4:30pm.

Assignment Policy: Late assignments will not be accepted. Students are encouraged to collaborate on assignments, but at the level of discussion only. When writing the solutions, they should do so in their own words. Past experience has shown conclusively that those who do not put adequate effort into the assignments do not learn the material and have a probability near 1 of doing poorly on the exams.

Important note: When writing your solutions, you must follow the guidelines below.

- The answers should be concise, clear and neat.
- When presenting proofs, every step should be justified.
- Assignments should be stapled or placed in an unsealed envelope.

Substantial departures from the above guidelines will not be graded.

Question 1: (**6+7+7 marks**) Give context-free grammars that generate the following languages. In all cases, the set Σ of terminals is equal to $\{0, 1\}$. For each case, justify your answer.

1. $\{w : w$ contains at least three 1s$\}$.
2. $\{w : w$ starts and ends with the same symbol$\}$.
3. $\{w :$ the length of w is odd and its middle symbol is $0\}$.

Question 2: (**20 marks**) Let $G = (V, \Sigma, R, S)$ be the context-free grammar, where $V = \{A, B, S\}$, $\Sigma = \{0, 1\}$, S is the start variable, and R consists of the rules

$$\begin{aligned} S &\rightarrow 0S|1A|\epsilon \\ A &\rightarrow 0B|1S \\ B &\rightarrow 0A|1B \end{aligned}$$

Define the following language L:

$$L := \{w \in \{0,1\}^* : \; w \text{ is the binary representation of a non-negative integer that is divisible by three } \} \cup \{\epsilon\}$$

Prove that $L = L(G)$. (*Hint:* The variables S, A, and B are used to remember the remainder after division by three.)

Question 3: (**6+7+7 marks**) Let A and B be context-free languages over the same alphabet Σ.

(3.1) Prove that the union $A \cup B$ of A and B is also context-free.

(3.2) Prove that the concatenation AB of A and B is also context-free.

(3.3) Prove that the star A^* of A is also context-free.

Question 4: (**10+10 marks**) Give (deterministic or nondeterministic) pushdown automata that accept the following languages.

1. $\{w \in \{0,1\}^* : w \text{ contains more 1s than 0s}\}$.

2. $\{w \in \{0,1\}^* : w \text{ is a palindrome}\}$. (A string w is a palindrome if $w = w^R$, i.e., reading w from left to right gives the same result as reading w from right to left. The empty string ϵ is a palindrome.)

Question 5: (**10+10 marks**) Prove that the following languages are not context-free:

1. $\{a^n b^n a^n b^n : n \geq 0\}$. The alphabet is $\{a, b\}$.

2. $\{w\#x : \ w \text{ is a substring of } x\text{; and } w, x \in \{a,b\}^*\}$.

   The alphabet is $\{a, b, \#\}$. The string $aba\#abbababbb$ is in the language, whereas the string $aba\#baabbaabb$ is not in the language.