COMP 2805 — Assignment 2

Due: February 12, before 23:59 pm, in the course drop box in Herzberg 4135 (there are two boxes having label 2805).

Assignment Policy: Late assignments will not be accepted. Students are encouraged to collaborate on assignments, but at the level of discussion only. When writing the solutions, they should do so in their own words. Past experience has shown conclusively that those who do not put adequate effort into the assignments do not learn the material and have a probability near 1 of doing poorly on the exams.

Important note: When writing your solutions, you must follow the guidelines below.

- The answers should be concise, clear and neat.
- When presenting proofs, every step should be justified.
- Assignments should be stapled or placed in an unsealed envelope.

Substantial departures from the above guidelines will not be graded.

Question 1: Give regular expressions describing the following languages. In all cases, the alphabet is $\{0, 1\}$.

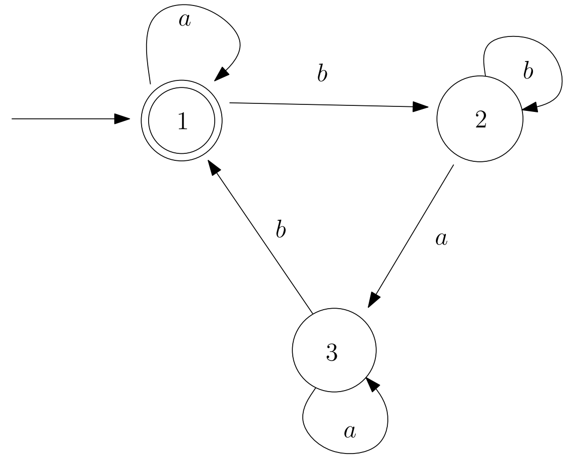
- $\{w : w$ contains at least five 1s$\}$.
- $\{w : w$ contains at least two 1s and at most one 0$\}$.
- $\{w : w$ contains an even number of 1s or exactly two 0s$\}$.

Question 2: Use the construction given in class (and described in the notes) to convert the regular expression

$$(((10)^*(00)) \cup 10)^*$$

to an NFA. The alphabet is $\{0, 1\}$.

Question 3: Use the construction given in class (and described in the notes) to convert the following DFA to a regular expression.



Question 4: Use the pumping lemma to prove that the following languages are not regular.

1. $\{a^n b^m c^{n+m} : n \geq 0, m \geq 0\}$.
2. $\{a^n b^m a^n : n \geq 0, m \geq 0\}$.
3. $\{a^{2^n} : n \geq 0\}$. (Remark: a^{2^n} is the string consisting of 2^n many a's.)
4. $\{a^n b^m c^\ell : n \geq 0, m \geq 0, \ell \geq 0, n^2 + m^2 = \ell^2\}$.

Question 5: Consider the language

$$A = \{a^i b^j c^j : i \geq 1, j \geq 0\} \cup \{b^j c^k : j \geq 0, k \geq 0\}.$$

Show that A satisfies the conclusion of the pumping lemma for $p = 1$. Thus, show that every string s in A whose length is at least one can be pumped.

(It can be shown that A is not a regular language. In other words, this example shows that the converse of the pumping lemma does, in general, not hold.)