COMP 2805 — Assignment 1

Due: January 29, before 23:59 pm, in the course drop box in Herzberg 4135 (there are two boxes having label 2805).

Assignment Policy: Late assignments will not be accepted. Students are encouraged to collaborate on assignments, but at the level of discussion only. When writing the solutions, they should do so in their own words. Past experience has shown conclusively that those who do not put adequate effort into the assignments do not learn the material and have a probability near 1 of doing poorly on the exams.

Important note: When writing your solutions, you must follow the guidelines below.

- The answers should be concise, clear and neat.
- When presenting proofs, every step should be justified.
- Assignments should be stapled or placed in an unsealed envelope.

Substantial departures from the above guidelines will not be graded.

Another important note: When specifying a finite automaton, you must give (i) the alphabet, (ii) the set of states, (iii) the start state, (iv) the set of accept states, and (v) the transition function. For each state that you use, you must describe its meaning in a clear way. The transition function can be described in the form of a state diagram.

Question 1: For each of the following languages, construct a DFA that accepts the language. In all cases, the alphabet is $\{0, 1\}$.
(1.1) $\{w : w$ contains at least five 1s$\}$.
(1.2) $\{w : w$ contains the substring 1011, i.e., $w = x1011y$ for some strings x and $y\}$.
(1.3) $\{w : w$ contains at least two 1s and at most two 0s$\}$.
(1.4) $\{w : w$ contains an odd number of 1s or exactly two 0s$\}$.

Question 2: For each of the following languages, construct an NFA, with the specified number of states, that accepts the language. In all cases, the alphabet is $\{0, 1\}$.
(2.1) The language $\{w : w$ ends with 10$\}$ with three states.
(2.2) The language $\{w : w$ contains the substring 1011$\}$ with five states.
(2.3) The language $\{w : w$ contains an odd number of 1s or exactly two 0s$\}$ with six states.

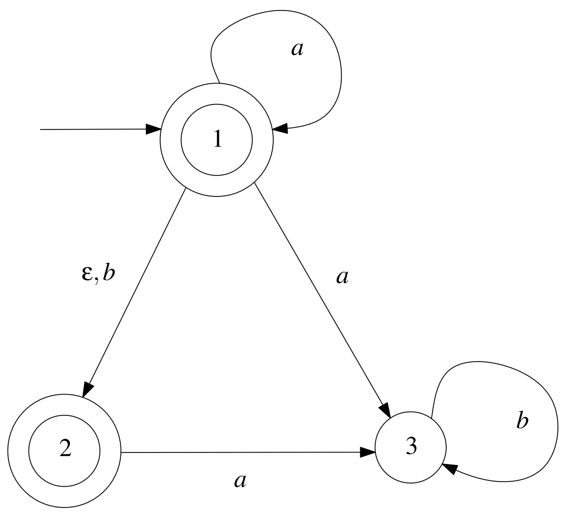
Question 3: Let A be a regular language. Prove that there exists an NFA that accepts A and that has exactly one accept state. (*Hint:* There exists a DFA/NFA that accepts A. If this automaton has more than one accept state, modify it.)

Question 4: For any string $w = w_1w_2 \dots w_n$, we denote by w^R the string obtained by reading w backwards, i.e., $w^R = w_nw_{n-1} \dots w_2w_1$. For any language A, we define A^R to be the language obtained by reading all strings in A backwards, i.e.,

$$A^R = \{w^R : w \in A\}.$$

Let A be a regular language. Prove that the language A^R is also regular. (*Hint:* Use the previous question.)

Question 5: Use the construction given in class (and described in the notes) to convert the following NFA to an equivalent DFA.



Question 6: Give regular expressions describing the following languages. In all cases, the alphabet is $\{0, 1\}$.

- $\{w : w$ contains at least five 1s$\}$.
- $\{w : w$ contains at least two 1s and at most one 0$\}$.
- $\{w : w$ contains an even number of 1s or exactly two 0s$\}$.